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term is $\frac{1}{r(n-r)}=\frac{1}{nr}+\frac{1}{n(n-r)}$, and since $\frac{1}{n-1}=\frac{1}{n}+\frac{1}{n(n-1)}$, $\frac{1}{2(n-2)}=\frac{1}{2n}+\frac{1}{n(n-2)}$, $\frac{1}{l(n-l)}=\frac{1}{ln}+\frac{1}{n(n-l)}\equiv\frac{1}{n(n-1)}+\frac{1}{n}$, the sum of (2)= $\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}\text{..............}\frac{1}{n}+[\frac{1}{n(n-1)}+\frac{1}{n(n-2)}+\text{.....}+\frac{1}{n}]$. The terms within and without the parenthesis are now plainly identical; consequently, $\frac{1}{2}[\frac{1}{n-1}+\frac{1}{2(n-2)}+\frac{1}{3(n-3)}+\text{.....}+\frac{1}{l(n-l)}]$ substituted for $\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}$ .... $\frac{1}{ln}$ in the right hand member of (1) will satisfy the equation.

II. Solution by R. D. CARMICHAEL, Hartselle, Alabama.

Represent the series of the first member by S_n. Then,

$$S_2=\frac{1}{1.3}+\frac{1}{2.4}+\frac{1}{3.5}+\frac{1}{4.6}\text{.....}=\tfrac{1}{2}[(1-\tfrac{1}{3})+(\tfrac{1}{2}-\tfrac{1}{4})+(\tfrac{1}{3}-\tfrac{1}{5})+(\tfrac{1}{4}-\tfrac{1}{6})\text{.....}]=\tfrac{1}{2}(1+\tfrac{1}{2}).$$

$$S_3=\frac{1}{1.4}+\frac{1}{2.5}+\frac{1}{3.6}+\text{.....}=\tfrac{1}{3}[(1-\tfrac{1}{4})+(\tfrac{1}{2}-\tfrac{1}{5})+(\tfrac{1}{3}-\tfrac{1}{6})+(\tfrac{1}{4}-\tfrac{1}{7})\text{.....}]=\tfrac{1}{3}(1+\tfrac{1}{2}+\tfrac{1}{3}).$$

....

$$S_n=\frac{1}{n}(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}\text{.....}+\frac{1}{n})=\frac{1}{n^2}+\frac{1}{n}(1+\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{4}\text{.....}+\frac{1}{n-1})$$

$$=\frac{1}{n^2}+\frac{1}{n}(\frac{1}{n-1}+\frac{1}{n-2}+\frac{1}{n-3}\text{.....}+\tfrac{1}{3}+\tfrac{1}{2}+1)$$

$$\therefore 2S_n=\frac{2}{n^2}+\frac{1}{n}[(1+\frac{1}{n-1})+(\tfrac{1}{2}+\frac{1}{n-2})+(\tfrac{1}{3}+\frac{1}{n-3})\text{.....}+(1+\frac{1}{n-1})].$$

$$\therefore S_n=\frac{1}{n^2}+\tfrac{1}{2}[\frac{1}{n-1}+\frac{1}{2(n-2)}+\frac{1}{3(n-3)}+\text{..........}\frac{1}{l(n-l)}],$$ l being equal to $n-1$.

Also solved by G. W. Greenwood, Henry Heaton, and G. B. M. Zerr.

252. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

Solve (1) $x-y=\frac{1}{3}\pi$; (2) $\sin x=\cos^3 y$.

Solution by J. SCHEFFER, Hagerstown, Md.

Since $x=y+\frac{1}{3}\pi$, we have $\sin x=\frac{1}{2}\sin y+\frac{1}{2}\sqrt{3}\cos y$.

$\therefore \frac{1}{2}\sin y+\frac{1}{2}\sqrt{3}\cos y=\cos^3 y$.

$\therefore 1-\cos^2 y=4\cos^6 y-4\sqrt{3}\cos^4 y+3\cos^2 y$, or $4\cos^6 y-4\sqrt{3}\cos^4 y+4\cos^2 y-1=0$; putting $\cos^2 y=z$, we get $z^3-\sqrt{3}z^2+z-\frac{1}{4}=0$; putting $z=t+\frac{1}{3}\sqrt{3}$, we get $t^3=\frac{1}{4}-\frac{1}{9}\sqrt{3}$; $\therefore t=(\frac{1}{4}-\frac{1}{9}\sqrt{3})^{\frac{1}{3}}$.

$$\therefore y=\cos^{-1}\pm[\tfrac{1}{3}\sqrt{3}+(\tfrac{1}{4}-\tfrac{1}{9}\sqrt{3})^{\frac{1}{3}}]^{\frac{1}{2}},\ x=\sin^{-1}\pm[\tfrac{1}{3}\sqrt{3}+(\tfrac{1}{4}-\tfrac{1}{9}\sqrt{3})^{\frac{1}{3}}]^{\frac{3}{2}}.$$

The two acute angles x and y are very nearly* $71^\circ\ 1'$ and $11^\circ\ 1'$.

Also solved by R. D. Carmichael, S. A. Corey, Henry Heaton, G. B. M. Zerr.

AVERAGE AND PROBABILITY.

174. Proposed by HENRY HEATON, Atlantic, Iowa.

Chords are drawn through every point of the surface of a given circle in every possible direction. What is their average length?

Solution by J. EDWARD SANDERS, Reinersville, Ohio.

Let θ=the angle the chord makes with r, then its length is $2\sqrt{(a^2-r^2\sin^2\theta)}$.

$$\therefore \Delta=\frac{4}{\pi a^2}\int_0^{\frac{1}{2}\pi}d\theta\int_0^a 2\sqrt{(a^2-r^2\sin^2\theta)}rdr=\frac{8a}{3\pi}\int_0^{\frac{1}{2}\pi}\frac{1-\cos^3\theta}{\sin^2\theta}d\theta=\frac{16a}{3\pi}.$$

Also solved by G. B. M. Zerr, and the Proposer.

175. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

If a line l is divided into three parts by two points taken at random on it, what is the mean value of the triangle whose sides are equal to the three parts? (Only those cases are to be considered in which the the three parts will form a triangle.)

Remark by S. A. COREY, Hiteman, Iowa.

Assuming that by mean value is meant average area, the problem becomes identical with problem 156, Average and Probability, a solution of which appeared in the MONTHLY in December, 1904, on page 237. The average area as there given is† $\pi l^2/105$.

CALCULUS.

213. Proposed by EDWIN L. RICH, Schenectady, N. Y.

Let $f(x)$ be any function of x, and $f'(x)$ its derivative. If $u=[f'(x)]^{-\frac{1}{2}}$, $v=f(x)[f'(x)]^{-\frac{1}{2}}$, then $\dfrac{1}{u}\dfrac{d^2u}{dx^2}-\dfrac{1}{v}\dfrac{d^2v}{dx^2}=0$.

I. Solution by W. L. TRYON, Ithaca, N. Y.

If $u=[f'(x)]^{-\frac{1}{2}}$, $\dfrac{du}{dx}=-\frac{1}{2}[f'(x)]^{-\frac{3}{2}}f''(x)$,

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*By the use of graphical methods Dr. Westlund obtains $x=71^\circ\ 1'\ 27''$, $y=11^\circ\ 1'\ 27''$. Mr. Heaton's result, correct to the sixth decimal place, is $x=71^\circ\ 1'\ 28''$, $y=11^\circ\ 1'\ 28''$. G.

†Solutions were contributed by Henry Heaton, J. Edward Sanders, and G. B. M. Zerr. The problem is solved in Williamson's *Integral Calculus*, Seventh Edition, p. 359. G.